# L(2,1) Graph Labelling

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# Overview

1. What's L(2,1) labeling?
2. Mathematical Bounds
3. Our Implementation

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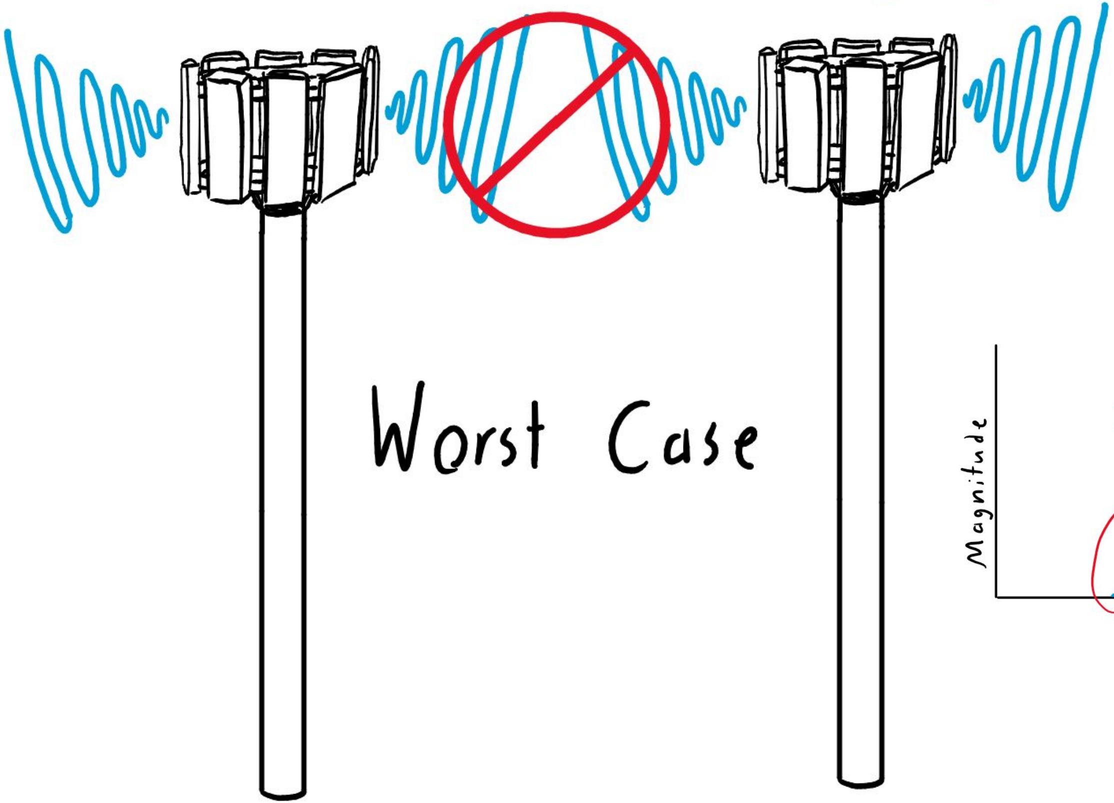
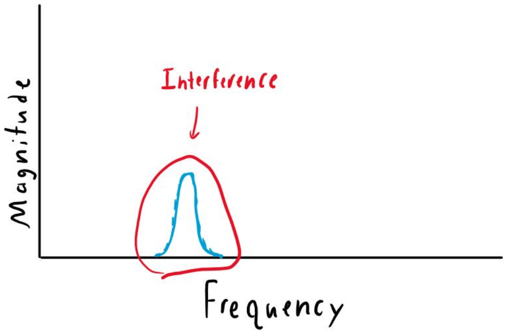
# What's L(2,1) Labeing?

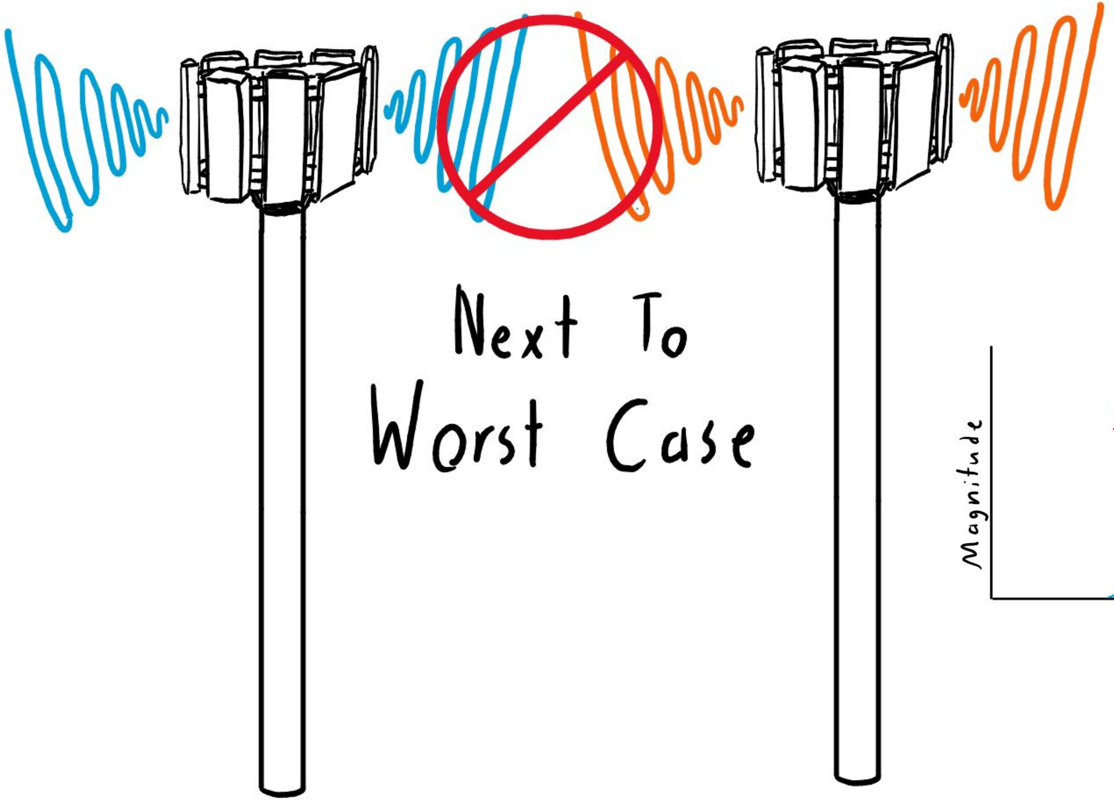
# Two Towers On Band Blue



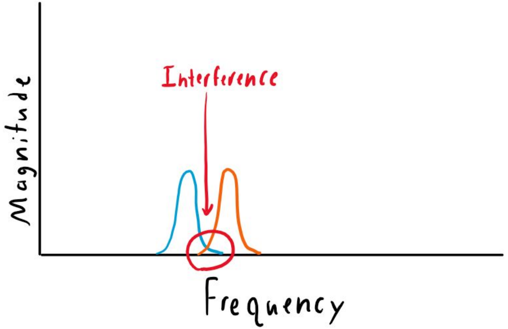
## Worst Case



# Two Towers On Adjacent Bands



Next To Worst Case



# L(2,1)-Labeling:

## A special graph coloring

For a graph, whenever x and y are two adjacent vertices then their label must have a distance greater than or equal to two.

Whenever x and y are two vertices with distance **two** between them, then their label must have a distance greater than or equal to **one**.

# Let's do an example!


Say we have a three vertex, two edge graph, and we want to assign them a minimum label from the set {0,1,2,3,4...}

# Let's do an example!

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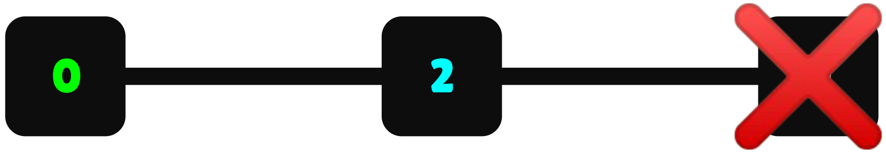
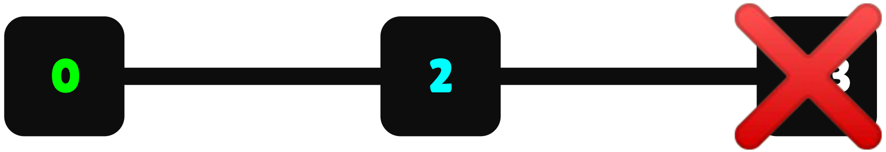
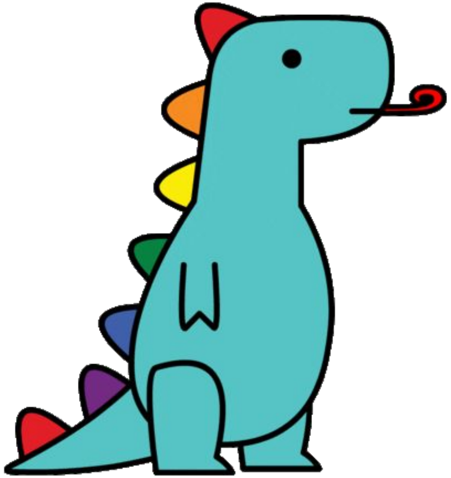


# Let's do an example!



# Done! But, we do have two holes...

# Mathematical Bounds: Minimizing the largest label

# Finding λ(G): The minimum span of labels required {0, 1, ... λ}

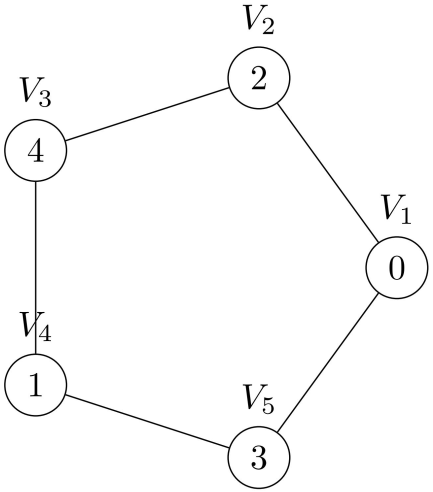
Griggs and Yeh's Conjectured Bound for λ(G)

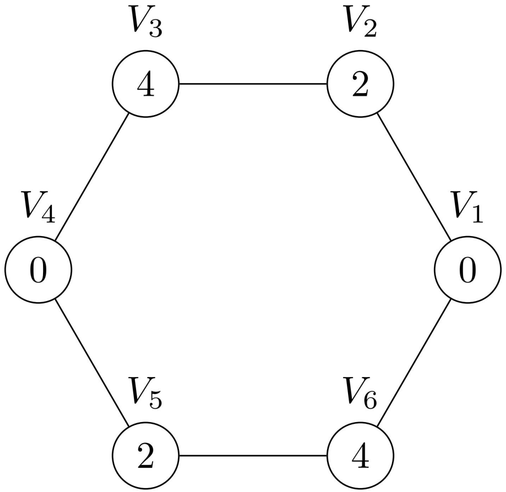
$$\lambda(G) \leq \Delta^2$$

Δ = Highest degree vertex in a graph G


$\Delta^2 = 4$

# Upper Bound for Cycles



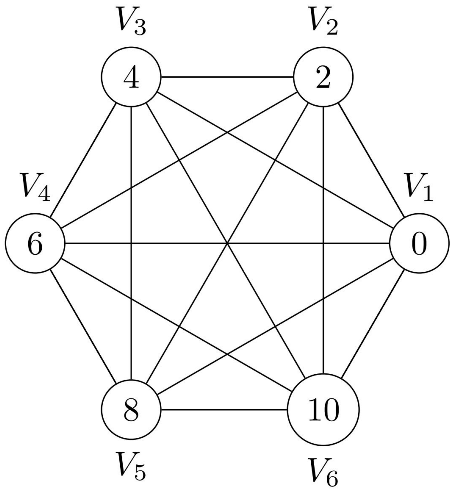




$\lambda = 4$ for C(n) where $n \geq 3$

Some cycles have holes; others do not

$\Delta = 2$, so cycles meet the conjectured $\Delta^2$ bound

# Upper Bound for Complete Graphs




λ = 2(n-1) for K(n) where n ≥ 3

Only even labeling numbers are used

Δ = n-1, so complete graphs meet the conjectured $\Delta^2$ bound

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## Our Implementation

# Algorithms for Minimizing λ(G)

Δ = Highest degree vertex in a graph G

1. Greedy Algorithm
   $\Delta^2+2\Delta$

2. Modified Chang-Kuo Algorithm
   $\Delta^2+\Delta-2$

3. Griggs and Yeh’s Conjectured Bound (No known algorithm)
   $\Delta^2$

# Algorithmic Differences

## Greedy Algorithm

- Iterate through vertices
- Assign lowest possible number
- Not seeing bigger picture, so prone to holes

## Modified Chang-Kuo Algorithm

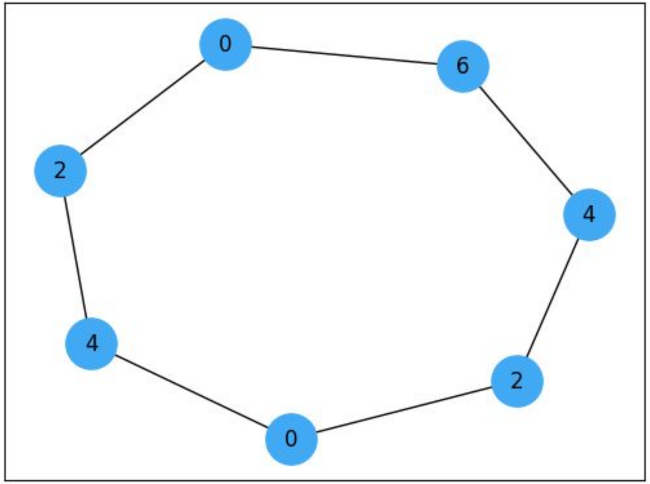
- Iterate through labeling numbers
- Assign current number to as many vertices as possible
- Looks at entire graph each time, so reduces likelihood of holes

# Our approach

1. Takes a list of edge connections defined by the user
2. Converts this into an adjacency matrix
3. Solves and plots the labeled graphs with either the greedy or modified Chang-Kuo algorithm

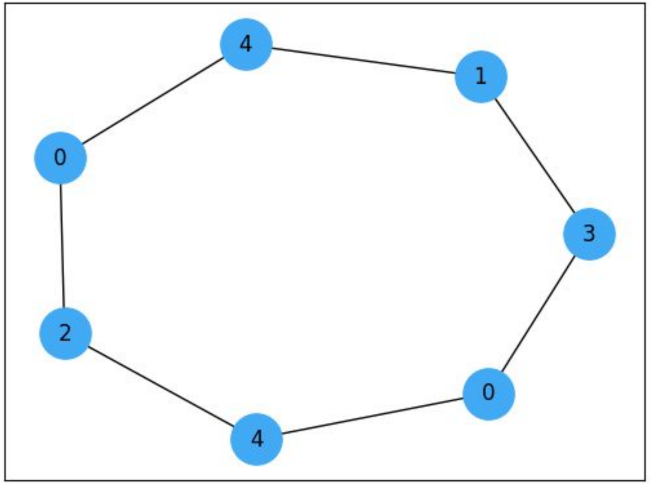
# Our Results (Cyclical Graph)

Greedy




λ = 6, Holes = 3

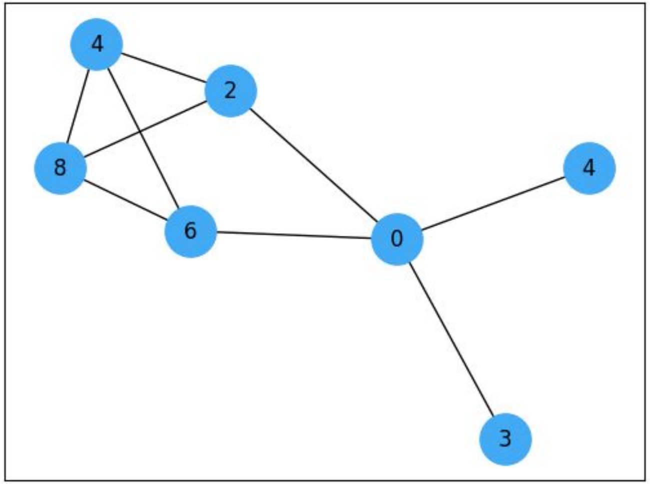
Chang-Kuo




λ = 4, Holes = 0

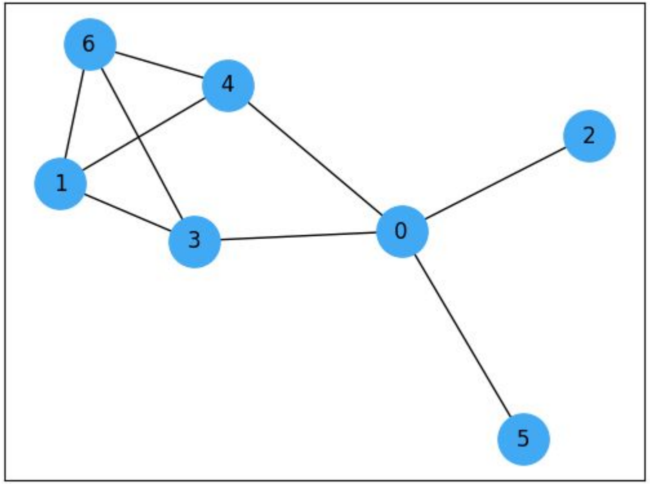
# Our Results (Random Graph)

Greedy




λ = 8, Holes = 3

Chang-Kuo




λ = 6, Holes = 0

# Check out our Code!

https://github.com/olincollege/L21-Graph-Coloring/tree/main

**Thank You!**
**Special Thanks to Professor**
**Nathaniel Karst of Babson (Olin ‘07)**
**And Sarah Spence Adams!**